8.1 Graphing Simple Rational Functions
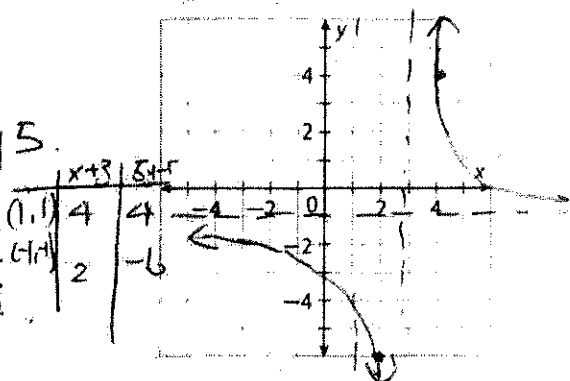
1. Tell the transformations that have been applied to the graph of $f(x) = \frac{1}{x}$ to produce the graph of $g(x)$. Give the equations of the asymptotes. Express the domain and range in set notation. Sketch the graph of $g(x)$. $g(x) = \frac{5}{x-3} - 1$

Transformations: _down 1, right 3, V.S by 5._

Vertical Asymptote: _x = 3_   Domain: _{x | x ≠ 3}_

Horizontal Asymptote: _y = 1_   Range: _{y | y ≠ 1}_

x+3	5÷5
(1,1) 4	4
(-1,-1) 2	-6

2. Rewrite the function in $g(x) = a\left(\frac{1}{(x-h)}\right) + k$ or $g(x) = \left(\frac{1}{\frac{1}{b}(x-h)}\right) + k$ form. Then tell the transformations that have been applied to the graph of $f(x) = \frac{1}{x}$ to produce the graph of $g(x)$. Give the equations of the asymptotes. Express the domain and range in set notation. Sketch the graph of $g(x)$.
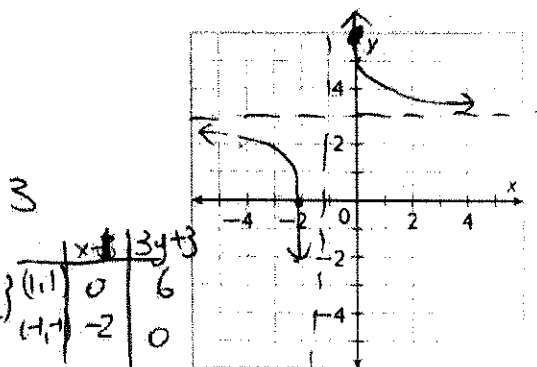
$g(x) = \frac{3x+6}{x+1}$

$$\begin{array}{r} 3 \\ x+1 \overline{)\, 3x+6} \\ \underline{3x+3} \\ 3 \end{array}$$

$3\frac{1}{x+1} + 3$

Transformations: _up 3 left 1 V.S. by 3_

Vertical Asymptote: _x = -1_   Domain: _{x | x ≠ -1}_

Horizontal Asymptote: _y = 3_   Range: _{y | y ≠ 3}_

x+1	3y+3
(1,1) 0	6
(-1,-1) -2	0

3. Write a function for the graph in the form $g(x) = a\left(\frac{1}{(x-h)}\right) + k$.

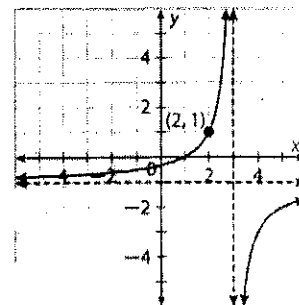$g(x) = a\left(\frac{1}{x-3}\right) - 1$

$1 = a\left(\frac{1}{2-3}\right) - 1$

$2 = a \cdot \frac{1}{-1}$

$a = -2$

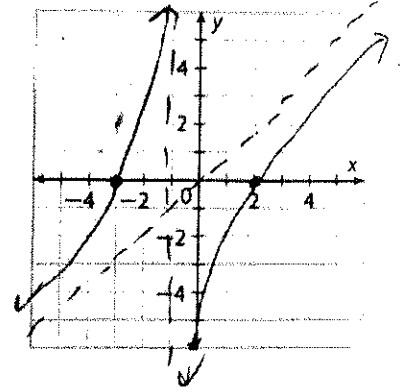$g(x) = 2\left(\frac{1}{x-3}\right) - 1$

8.2 Graphing More Complicated Rational Functions

For 4-6, graph each rational function by finding the vertical asymptote(s), any holes, horizontal or slant asymptote, and the x- and y-intercepts.

4. $f(x) = \frac{x^2 + x - 6}{x + 1}$

$$\frac{(x+3)(x-2)}{x+1}$$

$$x+1 \overline{\smash{\big)}\, x^2 + x - 6} \quad \overset{x}{}$$
$$\underline{x^2 + x}$$
$$-6$$

Vertical Asymptote(s): ___X = -1___   Holes: ___none___

Horizontal or Slant Asymptote: ___y = x___

x – intercept(s): ___-3, 2___   y – intercept: ___-6___

5. $f(x) = \frac{x + 3}{x^2 - x - 2}$

$$= \frac{x+3}{(x-2)(x+1)}$$

$x < -3$	$-$	$-$	$-$
$-3 < x < -1$	$+$	$-$	$+$
$-1 < x < 2$	$+$	$-$	$+$	$-$
$2 < x$	$+$	$+$	$+$	$+$
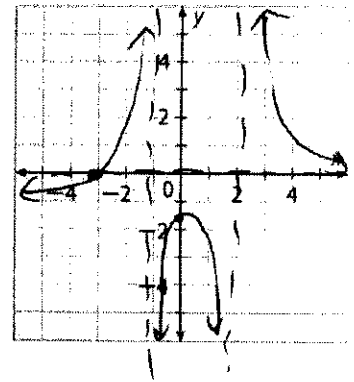
Vertical Asymptote(s): ___2, -1___   Holes: ___none___

Horizontal or Slant Asymptote: ___y = 0___

x – intercept(s): ___-3___   y – intercept: ___$\frac{3}{2}$___

6. $f(x) = \frac{-x^2 + 1}{x^2 - 3x + 2}$ $\frac{-(x+1)(x-1)}{(x-1)(x-2)} = \frac{-(x+1)}{x-2}$
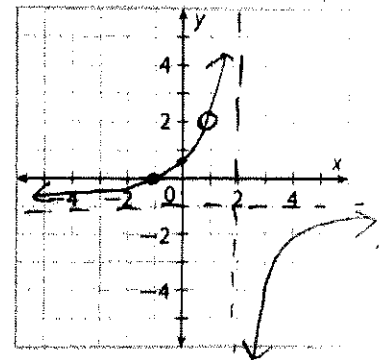
Vertical Asymptote(s): ___X = 2___   Holes: ___X = 1___

Horizontal or Slant Asymptote: ___y = -1___

x – intercept(s): ___X = -1___   y – intercept: ___½___

9.1 Adding and Subtracting Rational Expressions

Identify the excluded values for each expression.

1. $\dfrac{x-7}{9x^2-63x} = \dfrac{x-7}{4x(x-7)}$ $x \neq 0$
   $x \neq 7$
   $= \dfrac{1}{9x}$

2. $\dfrac{x^2+3x-18}{-x^2+6x-9}$ $\dfrac{(x+6)(x-3)}{(x-3)(x-3)}$ $x \neq 3$,
   $-(x^2-6x+9)$

Simplify the given expression and state any excluded values.

$2(x^2-6x+8)$

$\dfrac{2}{5}\ \dfrac{2}{4}$

3. $\dfrac{2x^2-12x+16}{7x^2-28x} = \dfrac{2(x-2)(x-4)}{7x(x-4)}$
   $= \dfrac{2(x-2)}{7x}$ $x \neq 0$
   $x \neq 4$

4. $\dfrac{5x^2+6x-8}{6x^2-24}$
   $\dfrac{(x+2)(5x-4)}{6(x+2)(x-2)}$
   $= \dfrac{5x-4}{6(x-2)}$ $x \neq 2$
   $x \neq -2$

5. $\dfrac{9x^3+9x^2}{7x^2-2x-9}$
   $= \dfrac{9x^2(x+1)}{(x+1)(7x-9)}$ $x \neq -1$
   $x \neq \dfrac{9}{7}$
   $\dfrac{1}{7}\ \dfrac{1}{-9}$
   $= \dfrac{9x^2}{7x-9}$

6. $\dfrac{2x^2+13x-24}{7x+56}$
   $\dfrac{2\ 8}{-3}$ $\dfrac{(x+8)(2x-3)}{7(x+8)}$
   $= \dfrac{2x-3}{7}$ $x \neq -8$

Add or Subtract. Identify any excluded values.

7. $\dfrac{2x-3}{x+4}+\dfrac{4x-5}{x+4}$
   $= \dfrac{6x-8}{x+4}$ $x \neq -4$

8. $\dfrac{x+12}{2x-5}-\dfrac{3x-2}{2x-5}$
   $= \dfrac{-2x+14}{2x-5}$ $x \neq \dfrac{5}{2}$

9. $\dfrac{x+4}{x^2-x-12}+\dfrac{2x}{x-4}$
   $\dfrac{x+4}{(x-4)(x+3)} + \dfrac{2x\ (x+3)}{x-4\ (x+3)}$
   $= \dfrac{x+4+2x^2+6x}{(x-4)(x+3)}$
   $= \dfrac{2x^2+7x+4}{(x-4)(x+3)}$ $x \neq 4$
   $x \neq -3$

10. $\dfrac{3x^2-1}{x^2-3x-18}-\dfrac{x+2}{x-6}$
    $\dfrac{3x^2-1}{(x-6)(x+3)} - \dfrac{x+2\ (x+3)}{x-6\ (x+3)}$
    $\dfrac{3x^2-1-(x^2+5x+6)}{(x-6)(x+3)}$
    $\dfrac{2x^2-5x-7}{(x-6)(x+3)}$ $x \neq 6$
    $x \neq -3$

11. $\dfrac{x+2}{x^2-2x-15} + \dfrac{x}{x+3}$

$\dfrac{x+2}{(x-5)(x+3)} + \dfrac{x}{x+3}\dfrac{(x-5)}{(x-5)}$

$\dfrac{x+2+x^2-5x}{(x+3)(x-5)}$

$\dfrac{x^2-4x+2}{(x+3)(x-5)}$ $x \neq -3$
   $x \neq 5$

12. $\dfrac{x+6}{x^2-7x-18} - \dfrac{2x}{x-9}$

$\dfrac{x+6}{(x-9)(x+2)} - \dfrac{2x}{x-9}\dfrac{(x+2)}{(x+2)}$

$\dfrac{x+6-2x^2-4x}{(x-9)(x+2)}$

$\dfrac{-2x^2-3x+6}{(x-9)(x+2)}$ $x \neq 9$
   $x \neq -2$

9.2 Multiplying and Dividing Rational Expressions

Multiply. Identify any excluded values.

1. $\dfrac{6x}{10} \cdot \dfrac{10}{8x^2} = \dfrac{6}{5x}$ $x \neq 0$

2. $\dfrac{4x}{3} \cdot \dfrac{8x}{2} = \dfrac{16x^2}{3}$

3. $\dfrac{1}{x+9} \cdot \dfrac{7x^3+49x^2}{x+7}$

$= \dfrac{1}{x+9} \cdot \dfrac{7x^2(x+7)}{x+7}$

$= \dfrac{7x^2}{x+9}$ $x \neq -7$
   $x \neq -9$

4. $\dfrac{6x^2-54x}{x-9} \cdot \dfrac{7x}{6x}$

$\dfrac{6x(x-9)}{x-9} \cdot \dfrac{7x}{6x} = 7x$ $x \neq 0$
   $x \neq 9$

5. $\dfrac{18x-36}{4x-8} \cdot \dfrac{2}{9x+18}$

$\dfrac{18(x-2)}{4(x-2)} \cdot \dfrac{2}{4(x+2)} = \dfrac{1}{x+2}$
   $x \neq 2$
   $x \neq -2$

6. $(56+11x-15x^2) \cdot \dfrac{10}{15x^2-11x-56}$
$\dfrac{-1}{(3x-7)(5x+8)}$ $\dfrac{3}{5}\ \dfrac{7}{8}$

$= -10$

   $x \neq \dfrac{7}{5}$

   $x \neq -\dfrac{8}{5}$

Divide. Identify any excluded values.

7. $\dfrac{4x}{5x} \div \dfrac{4x}{6}$

$\dfrac{4x}{5x} \cdot \dfrac{6}{4x} = \dfrac{6}{5x}$

   $x \neq 0$

8. $\dfrac{6(x-2)}{(x-1)(x-10)} \div \dfrac{x-2}{x-10}$

$\dfrac{6(x-2)}{(x-1)(x-10)} \cdot \dfrac{x-10}{x-2}$

$= \dfrac{6}{x-1}$ $x \neq 1$
   $x \neq 10$
   $x \neq 2$

9. $(2x+6) \div \dfrac{14x^2+42x}{10}$

$2x+6 \cdot \dfrac{10}{7x(2x+6)}$

$= \dfrac{10}{7x} \qquad x \neq 0$

$\qquad\qquad x \neq -3$

10. $\dfrac{27x+9}{10} \div \dfrac{3x^2-8x-3}{10}$

$\begin{array}{cc} 1 & -3 \\ 3 & 1 \end{array}$

$\dfrac{9(3x+1)}{10} \cdot \dfrac{10}{(x-3)(3x+1)}$

$= \dfrac{9}{x-3} \qquad x \neq 3$

$\qquad\qquad x \neq -\dfrac{1}{3}$

11. $\dfrac{24x+56}{10x^3-90x^2} \div \dfrac{15x+35}{5}$

$\dfrac{8(3x+7)}{10x^2(1x-9)} \cdot \dfrac{5}{5(3x+7)}$

$\dfrac{4}{5x^2(x-9)} \qquad x \neq 0 \quad x = -7/3$

$\qquad\qquad x \neq 9$

12. $\dfrac{2x+20}{12x^3-30x^2} \div \dfrac{2}{14x-35}$

$\dfrac{2(x+10)}{6x^2(2x-5)} \cdot \dfrac{7(2x-5)}{2}$
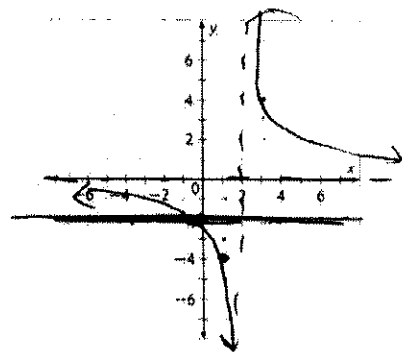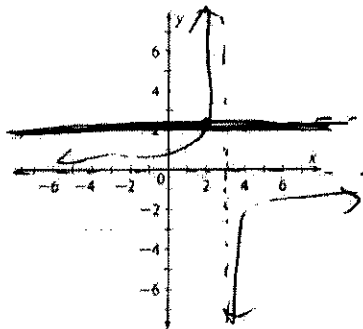
$\dfrac{7(x+10)}{6x^2} \qquad x \neq 0$

$\qquad\qquad x \neq \dfrac{5}{2}$

9.3 Solving Rational Equations

Graph to solve the equation.

1. $-\dfrac{2}{x-3} = 2$ ← Graphing Calculator → **2.** $\dfrac{4}{x-2} = -2$

$(2, 2)$ $\qquad\qquad\qquad\qquad\qquad\qquad (0, -2)$

Find the LCD for each pair.

3. $\dfrac{13}{4x}$ and $\dfrac{27}{3x^2}$

$12x^2$

4. $\dfrac{11}{x^2+3x+2}$ and $\dfrac{1}{x+2}$

$(x+1)(x+2)$

Solve each equation algebraically.

5. $\dfrac{\overset{3x}{1}}{x} - \dfrac{\overset{3x}{x-2}}{3x} = \dfrac{\overset{3x}{4}}{3x}$

$3 - x + 2 = 4$

$-x = -1$

$x = 1.$

6. $\dfrac{5x-5}{x^2-4x} - \dfrac{5}{x^2-4x} = \dfrac{1}{x}$ $x(x-4)$

$5x - 5 - 5 = x - 4$

$4x = 6$

$x = \dfrac{3}{2}$

7. $\dfrac{x^2-7x+10}{x} + \dfrac{1}{x} = (x+4)x$

$x^2 - 7x + 10 + 1 = x^2 + 4x$

$-11x = -11$

$x = 1$

8. $\dfrac{4}{x^2-4} = \dfrac{1}{x-2}$ $(x+2)(x-2)$

$4 = x + 2$

$2 = x$

no solution

Write a rational equation and solve.

9. The time required to deliver and install a computer at a customer's location is $t = 4 + \dfrac{d}{r}$, where t is time in hours, d is the distance, in miles, from the warehouse to the customer's location, and r is the average speed of the delivery truck. If it takes 6.2 hours for the employee to deliver and install a computer for a customer located 100 miles from the warehouse, what is the average speed of the delivery truck?

$100 = 4 + \dfrac{6.2}{r}$

$96 = \dfrac{6.2}{r}$

$96r = 6.2$

$r = \dfrac{6.2}{96} = 0.065$